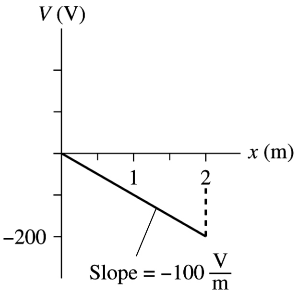
26

POTENTIAL AND FIELD

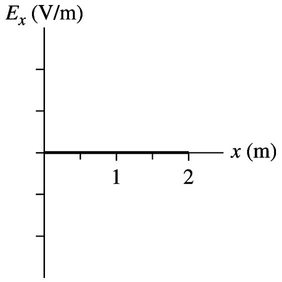
Conceptual Questions

26.1.



The electric field is the negative of the slope of the *V* vs. *x* graph.

26.2. The electric field is the negative of the slope of the *V* vs. *x* graph. If $V = \text{constant}$ in some region then $E = 0$ in that region.



26.3. **(a)** No. But you can conclude that $\Delta V = 0$ V; that is, the potential is constant in that region of space.

(b) Yes. When $V = \text{constant}$ in a region of space then $\vec{E} = \vec{0}$ V/m, since $E = -dV/ds$.

26.4.

$$\vec{E}_1 = \frac{-\Delta V}{\Delta s}\hat{i} = -\frac{10\ \text{V} - 0\ \text{V}}{1\ \text{m} - 0\ \text{m}} = -\frac{10\ \text{V}}{1\ \text{m}}\hat{i} = -(10\ \text{V/m})\hat{i}$$

$$\vec{E}_2 = \frac{-\Delta V}{\Delta s}\hat{i} = -\frac{40\ \text{V} - 30\ \text{V}}{4\ \text{m} - 3\ \text{m}} = -\frac{10\ \text{V}}{1\ \text{m}} = -(10\ \text{V/m})\hat{i}$$

Both fields point left, in the direction of decreasing potential.

26.5.

$$\vec{E}_1 = \frac{-\Delta V}{\Delta s}\hat{i} = -\frac{10\ \text{V} - 0\ \text{V}}{2\ \text{m} - 0\ \text{m}}\hat{i} = -(5\ \text{V/m})\hat{i}$$

$$\vec{E}_2 = \frac{-\Delta V}{\Delta s}\hat{i} = -\frac{40\ \text{V} - 20\ \text{V}}{4\ \text{m} - 3\ \text{m}} = -(20\ \text{V/m})\hat{i}$$

Both fields point left, in the direction of decreasing potential.

26.6. The electron moves to the right. The electric field is the negative of the slope of the electric potential, so the electric field will be nonzero at $x = 2$ m and point to the left. The electron will therefore experience a net force to the right and accelerate to the right.

26.7. (a) The electric field vector points in the direction of decreasing potential. Therefore $V_\text{a} > V_\text{b}$.

(b) $|\Delta V_\text{ab}| < |\Delta V_\text{cd}| < |\Delta V_\text{ef}|$. For a uniform electric field $|\Delta V| = |-E\Delta s| = |E\Delta s|$. The order is determined by the fact that $\Delta s_\text{ab} < \Delta s_\text{cd} < \Delta s_\text{ef}$.

(c) Surface 1 is an equipotential surface because it is perpendicular to the electric field vectors. Surface 2 is not perpendicular to the electric field so it is not an equipotential surface.

26.8. Equal. When a conductor is in electrostatic equilibrium, the entire conductor is at the same potential.

26.9. (a) $V_1 = V_2$. Both spheres and the wire become one conductor and so are all at the same potential.

(b) Since $V_1 = V_2$, $\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2}$, thus $\frac{Q_1}{r_1} = \frac{Q_2}{r_2}$. Since $r_1 > r_2$, $Q_1 > Q_2$.

(c) Recall $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2} = \frac{V_1}{r_1}$ and $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2} = \frac{V_2}{r_2}$. Since $r_1 > r_2$, $E_1 < E_2$.

26.10. Conducting wires are equipotentials if no current is flowing. The top wire is at the same potential as the top of the battery and the bottom wire is at the same potential as the bottom of the battery. Therefore,

$$\Delta V_{12} = 0\ \text{V} \qquad \Delta V_{23} = 3\ \text{V} \qquad \Delta V_{34} = 0\ \text{V} \qquad \Delta V_{14} = 3\ \text{V}$$

26.11. (a) No, ΔV_C does not change because the upper plate is still at the potential of the positive electrode of the battery because the two are connected by a conducting wire. Likewise, the bottom plate is connected to the negative electrode and so its potential remains unchanged. Therefore, the potential difference between the top and bottom plates remains unchanged.

(b) Yes. $C = \varepsilon_0 A/d$, so when the plate separation *d* is doubled the capacitance decreases by a factor of 2.

(c) Yes. The charge also decreases by a factor of 2. Because $C = Q/\Delta V_\text{C}$ decreases and ΔV_C remains constant (see part a), *Q* decreases by the same factor as the capacitance.

26.12. Calculate $\Delta V_C = Q/C$ for each capacitor:

$$(\Delta V_C)_1 = \frac{Q}{C} \quad (\Delta V_C)_2 = \frac{Q}{2C} = \frac{1}{2}(\Delta V_C)_1 \quad (\Delta V_C)_3 = \frac{2Q}{C} = 2(\Delta V_C)_1 \quad (\Delta V_C)_4 = \frac{2Q}{2C} = (\Delta V_C)_1$$

Thus, $(\Delta V_C)_3 > (\Delta V_C)_1 = (\Delta V_C)_4 > (\Delta V_C)_2$.

Exercises and Problems

Exercises

Section 26.1 Connecting Potential and Field

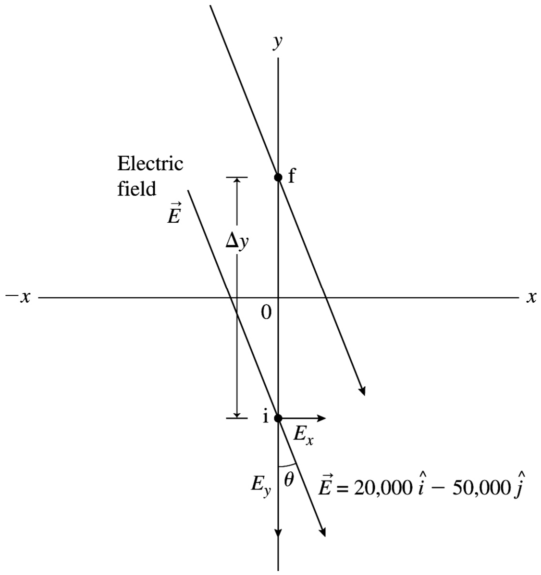
26.1. Solve: Apply Equation 26.3. Because we are working in one dimension and $\vec{E}$ is parallel to $d\vec{s}$, the dot product $\vec{E} \cdot d\vec{s} = Edx$. The potential difference ΔV between two points in space is

$$\Delta V = V(x_\text{f}) - V(x_\text{i}) = -\int_{x_\text{i}}^{x_\text{f}} E_x \, dx$$

where x is the position along a line from point i to point f. When the electric field is uniform,

$$\Delta V = -E_x \int_{x_\text{i}}^{x_\text{f}} dx = -E_x \Delta x = -(1000\ \text{V/m})(0.30\ \text{m} - 0.10\ \text{m}) = -200\ \text{V} = -0.20\ \text{kV}$$

26.2. Visualize:



Solve: The potential difference ΔV between two points on the y-axis is

$$\Delta V = -\int_{y_\text{i}}^{y_\text{f}} \vec{E} \cdot d\vec{s} = -\int_{y_\text{i}}^{y_\text{f}} E_y dy$$

When the electric field is uniform, the above result simplifies to $\Delta V = -E_y \Delta y$. In the present problem,

$$\Delta y = y_\text{f} - y_\text{i} = 0.05\ \text{m} - (-0.05\ \text{m}) = 0.10\ \text{m}$$

The y-component of the electric field is

$$E_y = -50{,}000\ \text{V/m} \Rightarrow \Delta V = -(-50{,}000\ \text{V/m})(0.10\ \text{m}) = +5.0\ \text{kV}$$

Assess: $V_\text{f} - V_\text{i} = 5.0\ \text{kV}$ shows that the potential at point f is higher than at point i. This is because the electric field is directed from the point at the higher potential to the point at the lower potential.

26.3. Model: The potential difference is the negative of the area under the E_x vs. x curve.
Visualize: Count the area below the x-axis as negative and the area above as positive.
Solve: The net area under the curve between $x = 1.0$ m and $x = 3.0$ m is -100 V, so $\Delta V = 100$ V.

26.4. Model: The potential difference is the negative of the area under the E_x vs. x curve.
Visualize: Please refer to Figure EX26.4.
Solve: The potential difference between the origin and $x = 3.0$ m is

$$\begin{aligned}\Delta V = V(x = 3.0\ \text{m}) - V(x = 0.0\ \text{m}) &= -\left[\tfrac{1}{2}(-100\ \text{V})(1.0\ \text{m} - 0\ \text{m}) + \tfrac{1}{2}(200\ \text{V})(3.0\ \text{m} - 1.0\ \text{m})\right] \\ &= -150\ \text{V}\end{aligned}$$

Thus $V(3.0\ \text{m}) = V(0\ \text{m}) - 150\ \text{V} = -50\ \text{V} - 150\ \text{V} = -200\ \text{V} = -0.20\ \text{kV}$
Assess: The potential decreases from the origin to $x = 3.0$ m.

26.5. Visualize: Please refer to Figure EX26.5.
Solve: (a) The electric field points "downhill." So, point A is at a higher potential than point B.
(b) Because electric field lines are everywhere perpendicular to equipotential lines, we only need consider the distance between the equipotential line that goes through point A and the equipotential line that goes through point B. This distance is 7 cm. In a region that has a uniform electric field, Equation 26.3 gives the potential difference between two points:

$$\Delta V = -\int E_x dx = -E_x(x_\text{f} - x_\text{i}) \Rightarrow V_\text{B} - V_\text{A} = -(1000\ \text{V/m})(0.07\ \text{m}) = -70\ \text{V}$$

That is, the potential at point A is 70 V higher than the potential at point B.

Section 26.2 Finding the Electric Field from the Potential

26.6. Model: The field is uniform between the plates.
Visualize: The potential is visualized as a sloping ramp.
Solve:

$$E_s = -\frac{\Delta V}{\Delta s} = -\frac{35\ \text{V} - 20\ \text{V}}{2.5\ \text{cm}} = -\frac{15\ \text{V}}{0.025\ \text{m}} = -600\ \text{V/m}$$

So the field strength is 600 V/m.
Assess: This is a typical field strength.

26.7. Model: The electric field points in the direction of *decreasing* potential and is perpendicular to the equipotential lines.
Visualize: Please refer to Figure EX26.7. The three equipotential surfaces correspond to potentials of 0 V, 200 V, and 400 V.
Solve: The electric field component along a direction of constant potential is $E_\text{s} = -dV/ds = 0$ V/m. But, the electric field component perpendicular to the equipotential surface is

$$\left|\vec{E}\right| = \frac{\Delta V}{\Delta s} = \frac{400\ \text{V}}{0.02\ \text{m}} = 20\ \text{kV/m}$$

The direction of the electric field vector is "downhill," perpendicular to the equipotential surfaces. That is, the electric field is 20 kV/m pointing downward.

26.8. Model: The electric field is perpendicular to the equipotential lines and points "downhill."
Visualize: Please refer to Figure EX26.8. Three equipotential surfaces at potentials of –200 V, 0 V, and 200 V are shown.
Solve: The electric field is perpendicular to the equipotential surfaces:

$$E = -\frac{\Delta V}{\Delta s} = -\frac{400\ \text{V}}{0.02\ \text{m}} = -20\ \text{kV/m}$$

The electric field vector is in the third quadrant, 45° below the negative x-axis. That is,

$$\vec{E} = (20\ \text{kV/m},\ 45°\ \text{below}\ -x\text{-axis})$$

Assess: In component form, this would be $\vec{E} = \left(-\frac{20}{\sqrt{2}}\hat{\imath} - \frac{20}{\sqrt{2}}\hat{\jmath}\right)$ kV/m.

26.9. Model: The electric field is the negative of the slope of the graph of the potential function.
Visualize: Please see Figure EX26.9.
Solve: (a) At $x = -2$ cm, the slope of the potential curve is

$$dV/dx = \frac{-10\ \text{V}}{2\ \text{m}} = -5\ \text{V/m}$$

so the electric field is $E = 5$ V/m.
(b) At $x = -0$ cm, the slope of the potential curve is

$$dV/dx = \frac{20\ \text{V}}{2\ \text{m}} = 10\ \text{V/m}$$

so the electric field is $E = -10$ V/m.
(c) At $x = 2$ cm, the slope of the potential curve is

$$dV/dx = \frac{-10\ \text{V}}{2\ \text{m}} = -5\ \text{V/m}$$

so the electric field is $E = 5$ V/m.

26.10. Model: The electric field is the negative of the slope of the potential graph.
Visualize: Please refer to Figure EX26.10.
Solve: Since the contours are uniformly spaced along the y-axis above and below the origin, the slope method is the easiest to apply. Point 1 is in the center of a 75 V change (25 V to 100 V) over a distance of 2 cm, so the slope $\Delta V/\Delta s$ is 37.5 V/cm or 3750 V/m. Point 2 has the same potential difference in half the distance. Thus the slope at point 2 is 7500 V/m. Thus, the magnitudes of the electric fields at points 1 and 2 are 3750 V/m and 7500 V/m. The directions of the electric fields are downward at point 1 and upward at point 2, that is, from the higher potential to the lower potential. All together, this gives

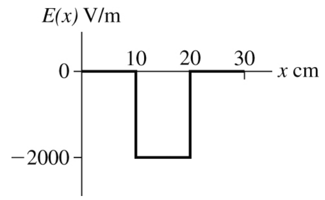
$$\vec{E}_1 = (3750\ \text{V/m, down}) \qquad \vec{E}_2 = (7500\ \text{V/m, up})$$

26.11. Model: The electric field is the negative of the slope of the graph of the potential function.
Visualize: Please refer to Figure EX26.11.
Solve: There are three regions of different slope. For $0\ \text{cm} \le x < 10\ \text{cm}$ and $20\ \text{cm} \le x < 30\ \text{cm}$,

$$\frac{\Delta V}{\Delta x} = 0\ \text{V/m} \Rightarrow E_x = 0\ \text{V/m}$$

For $10\ \text{cm} \le x < 20\ \text{cm}$,

$$\frac{\Delta V}{\Delta x} = \frac{100\ \text{V} - (-100\ \text{V})}{0.20\ \text{m} - 0.10\ \text{m}} = 2000\ \text{V/m} \Rightarrow E_x = -2000\ \text{V/m}$$



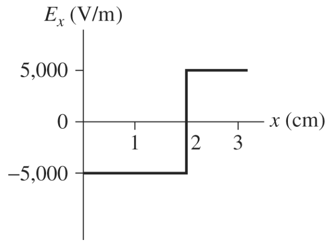
Assess: Because $E_s = -dV/ds,$ the electric field is zero where the potential is not changing.

26.12. Model: The electric field is the negative of the slope of the graph of the potential function.
Solve: There are two regions of different slope. For $0\text{ cm} \le x < 2\text{ cm},$

$$\frac{\Delta V}{\Delta x} = \frac{50\text{ V} - (-50\text{ V})}{0.02\text{ m } - 0\text{ m}} = 5000\text{ V/m} \Rightarrow E_x = -5000\text{ V/m}$$

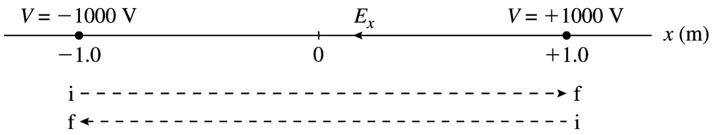
For $x \ge 2$ cm,

$$\frac{\Delta V}{\Delta x} = \frac{0\text{ V} - 50\text{ V}}{0.03\text{ m} - 0.02\text{ m}} = -5000\text{ V/m} \Rightarrow E_x = 5000\text{ V/m}$$



Assess: $\Delta V/\Delta x$ and E_x are the negative of each other.

26.13. Visualize:



Solve: The electric potential difference ΔV between two points in a uniform electric field is

$$V(x_f) - V(x_i) = -\int E_x dx = -E_x(x_f - x_i)$$

Choosing $x_\text{i} = -1.0$ m and $x_\text{f} = +1.0$ m,

$$+1000\text{ V} - (-1000\text{ V}) = -E_x[1.0\text{ m} - (-1.0\text{ m})] \Rightarrow E_x = -1.0\text{ kV/m}$$

Alternatively, $x_\text{i} = -1.0$ m and $x_\text{f} = -1.0$ m. For this choice,

$$-1000\text{ V} - (+1000\text{ V}) = -E_x[-1.0\text{ m} - (1.0\text{ m})] \Rightarrow E_x = -1.0\text{ kV/m}$$

Assess: The choice of initial and final positions does not change the physical nature of the electric field or the potential difference.

26.14. Model: The electric field is the negative of the derivative of the potential function.
Solve: **(a)** From Equation 26.9, the component of the electric field in the *s*-direction is $E_s = -dV/ds.$ For the given potential,

$$\frac{dV}{dx} = \frac{d}{dx}(100x^2 \text{ V}) = 200x \text{ V/m} \Rightarrow E_x = -200x \text{ V/m}$$

At $x = 0$ m, $E_x = 0$ V/m.

(b) At $x = 1$ m, $E_x = -200 \times 1 \text{ V/m} = -0.2 \text{ kV/m}$.

Assess: The potential increases with *x*, so the electric field must point in the –*x*-direction.

26.15. Solve: (a) Since $E_x = -dV/dx$, we have

$$E_x = -\frac{d}{dx}(100e^{-2x}) \text{ V/m} = 200e^{-2x} \text{ V/m}$$

At $x = 1.0$ m,

$$E_x = (200e^{-2(1.0 \text{ m})}) \text{ V/m} = 27 \text{ V/m}$$

(b) At $x = 2.0$ m,

$$E_x = (200e^{-2(2.0 \text{ m})}) \text{ V/m} = 3.7 \text{ V/m}$$

26.16. Model: Use Kirchhoff's loop law.
Visualize: Please refer to Figure EX26.16.
Solve: For any path that starts and ends at the same point, $\Delta V_{\text{loop}} = \sum_{\text{i}}(\Delta V)_{\text{i}} = 0$ V. Therefore,

$$\Delta V_{12} + \Delta V_{23} + \Delta V_{34} + \Delta V_{41} = 30 \text{ V} + 50 \text{ V} + \Delta V_{34} - 60 \text{ V} = 0 \text{ V} \Rightarrow \Delta V_{34} = -20 \text{ V}$$

That is, the potential at point 4 is less than the potential at point 3.

Section 26.4 Sources of Electric Potential

26.17. Solve: The work done is exactly equal to the increase in the potential energy of the charge. That is,

$$W = \Delta U = q\Delta V = q(V_{\text{f}} - V_{\text{i}}) = (1.0 \times 10^{-6} \text{ C})(1.5 \text{ V}) = 1.5 \times 10^{-6} \text{ J}$$

Assess: The work done by the escalator on the charge is stored as electric potential energy of the charge.

26.18. Solve: The work done is the change in potential energy of the charge, and the electric potential is the potential energy per unit charge, so

$$W = \Delta U = q\Delta V \Rightarrow q = \frac{W}{\Delta V} = \frac{27 \text{ J}}{9.0 \text{ V}} = 3.0 \text{ C}$$

26.19. Solve: The Van de Graaff generator (or the motor that runs the belt) does work in lifting a positive ion $(q = e)$ against the downward force on the positive charge. The work done is

$$W = \Delta U = q\Delta V = e(1.0 \times 10^6 \text{ V} - 0 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1.0 \times 10^6 \text{ V}) = 1.6 \times 10^{-13} \text{ J}$$

Assess: The work done by the generator in lifting the charge is stored as electric potential energy of the charge.

26.20. Solve: The work done is the potential energy gained by the electron.

$$W = U = q\Delta V = -e\Delta V$$

$$\Delta V = \frac{W}{-e} = \frac{(2.4 \times 10^{-19} \text{ J})}{-(1.6 \times 10^{-19} \text{ C})} = -1.5 \text{ V}$$

The potential difference from the positive to the negative terminal is –1.5 V, so the emf of the solar cell is 1.5 V.

Section 26.5 Capacitance and Capacitors

26.21. Model: Assume that the battery is ideal and that the capacitor is a parallel-plate capacitor.
Solve: (a) From Equation 26.18, the capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)\pi(0.015\ \text{m})^2}{0.00050\ \text{m}} = 1.25\times10^{-11}\ \text{F} = 13\ \text{pF}$$

(b) The magnitude of the charge on each electrode is

$$Q = C\Delta V_\text{c} = (1.25\times10^{-11}\ \text{F})(100\ \text{V}) = 1.25\times10^{-9}\ \text{C} = 1.3\ \text{nC}$$

26.22. Visualize: Please refer to Figure EX26.22.
Solve: Any two electrodes, regardless of their shape, form a capacitor whose capacitance is defined as $C = Q/\Delta V_\text{C}$. Therefore, the capacitance is

$$C = \frac{20\times10^{-9}\ \text{C}}{100\ \text{V}} = 2.0\times10^{-10}\ \text{F} = 0.20\ \text{nF}$$

26.23. Solve: From Equation 26.18, the capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 L^2}{d} \Rightarrow L = \sqrt{\frac{Cd}{\varepsilon_0}} = \sqrt{\frac{(100\times10^{-12}\ \text{F})(0.20\times10^{-3}\ \text{m})}{8.85\times10^{-12}\ \text{C}^2/\text{N m}^2}} = 0.048\ \text{m} = 4.8\ \text{cm}$$

Assess: The capacitance depends only on the geometry of the capacitor.

26.24. Model: Assume that the battery is ideal and the connecting wires have zero resistance.
Solve: The capacitance of a capacitor is the ratio of the charge and the potential difference ΔV_C:

$$C = \frac{Q}{\Delta V_\text{C}} \Rightarrow \Delta V_\text{C} = \frac{Q}{C} = \frac{30\times10^{-6}\ \text{C}}{10\times10^{-6}\ \text{F}} = 3.0\ \text{V}$$

For an ideal battery, $\varepsilon = \Delta V_\text{bat} = \Delta V_\text{C}$. So, $\varepsilon = 3.0$ V.

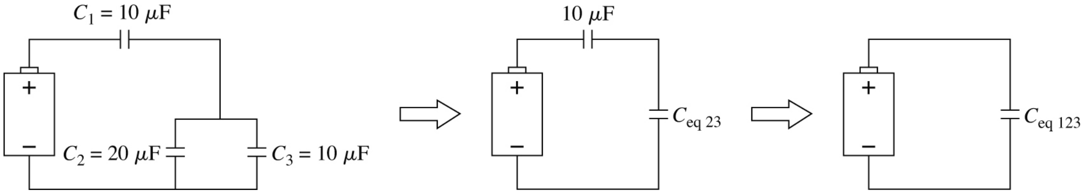
26.25. Solve: According to Equation 26.22,

$$C_\text{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{6\ \mu\text{F}} + \frac{1}{10\ \mu\text{F}} + \frac{1}{16\ \mu\text{F}}\right)^{-1} = 3.0\times10^{-6}\ \text{F} = 3.0\ \mu\text{F}$$

26.26. Solve: According to Equation 26.20,

$$C_\text{eq} = C_1 + C_2 + C_3 = 6\ \mu\text{F} + 10\ \mu\text{F} + 16\ \mu\text{F} = 32\ \mu\text{F}$$

26.27. Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.
Solve: Because C_2 and C_3 are in parallel, their equivalent capacitance $C_\text{eq 23}$ is

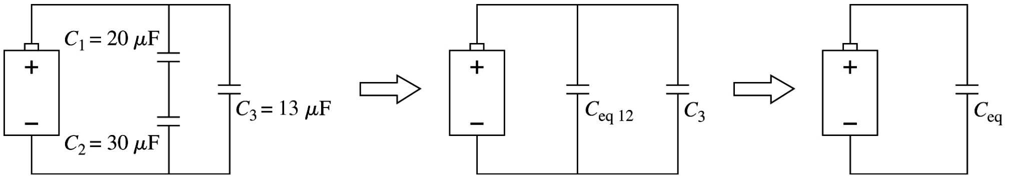
$$C_\text{eq 23} = C_2 + C_3 = 20\mu\text{F} + 10\mu\text{F} = 30\mu\text{F}$$

Then, $C_{\text{eq }23}$ and C_1 are in series. So,

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_{\text{eq }12}}+\frac{1}{C_3}=\frac{1}{30\ \mu\text{F}}+\frac{1}{10\ \mu\text{F}}=\frac{4}{30\ \mu\text{F}}\Rightarrow C_{\text{eq}}=\frac{30}{4}\ \mu\text{F}=7.5\ \mu\text{F}$$

Assess: For capacitors in series, the equivalent capacitance is less than any of the individual capacitors. In this case, $C_{\text{eq}}<30\ \mu\text{F}$ and $C_{\text{eq}}<10\ \mu\text{F}$, as expected.

26.28. Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.
Solve: Because C_1 and C_2 are in series, their equivalent capacitance $C_{\text{eq }12}$ is

$$\frac{1}{C_{\text{eq }12}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{20\ \mu\text{F}}+\frac{1}{30\ \mu\text{F}}=\frac{1}{12\ \mu\text{F}}\Rightarrow C_{\text{eq }12}=12\ \mu\text{F}$$

Then, $C_{\text{eq }12}$ and C_3 are in parallel. So,

$$C_{\text{eq}}=C_{\text{eq }12}+C_3=12\mu\text{F}+13\mu\text{F}=25\mu\text{F}$$

26.29. Model: Two capacitors in parallel combine to give greater capacitance.
Solve: Since we want a capacitance of $50\ \mu\text{F}$ and we have a $30\ \mu\text{F}$ capacitor, we must connect the second capacitor in parallel with the $30\ \mu\text{F}$ capacitor. That is,

$$C+30\ \mu\text{F}=50\ \mu\text{F}\Rightarrow C=50\ \mu\text{F}-30\ \mu\text{F}=20\ \mu\text{F}$$

26.30. Model: Two capacitors in series combine to give less capacitance.
Solve: Since we have a $75\ \mu\text{F}$ capacitor and we want a $50\ \mu\text{F}$ capacitance, we must connect the second capacitor in series with the $75\ \mu\text{F}$ capacitor. The capacitance of the second capacitor is calculated as follows:

$$\frac{1}{75\ \mu F}+\frac{1}{C}=\frac{1}{50\ \mu F}\Rightarrow C=150\ \mu\text{F}$$

Section 26.6 The Energy Stored in a Capacitor

26.31. Solve: From Equation 26.25, the energy stored in a capacitor is

$$U_{\text{C}}=\tfrac{1}{2}C(\Delta V_{\text{C}})^2\Rightarrow\Delta V_{\text{C}}=\sqrt{\frac{2U_{\text{C}}}{C}}=\sqrt{\frac{2(1.0\text{ J})}{1.0\times10^{-6}\text{ F}}}=1.4\text{ kV}$$

26.32. Solve: The energy density is $u_{\text{E}}=(50\times10^{-12}\text{ J})/(0.020\text{ m}\times0.020\text{ m}\times0.020\text{ m})=6.25\times10^{-6}\text{ J/m}^3$.

$$u_{\text{E}}=\tfrac{1}{2}\varepsilon_0E^2\Rightarrow E=\sqrt{\frac{2u_{\text{E}}}{\varepsilon_0}}=\sqrt{\frac{2(6.25\times10^{-6}\text{ J/m}^3)}{8.85\times10^{-12}\text{ C}^2\text{/N m}^2}}=1.2\text{ kV/m}$$

26.33. Solve: (a) From Equation 26.25, the energy stored in the charged capacitor is

$$U_{\text{C}} = \frac{1}{2}C(\Delta V_{\text{C}})^2 = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)(\Delta V_{\text{C}})^2$$

$$= \frac{1}{2}\frac{\pi(0.010\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)}{0.50\times10^{-3}\text{ m}}(200\text{ V})^2 = 1.1\times10^{-7}\text{ J}$$

(b) From Equation 26.27, the energy density in the electric field is

$$u_{\text{E}} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{\Delta V_{\text{C}}}{d}\right)^2 = \frac{1}{2}(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)\left(\frac{200\text{ V}}{0.50\times10^{-3}\text{ m}}\right)^2 = 0.71\text{ J/m}^3$$

26.34. Solve: The energy total energy discharged into the patient is $U_{\text{C}} = Pt = (6500\text{ W})(5.0\times10^3\text{ s}) = 32.5\text{ J}$. This energy was stored in the capacitor and may be found from Equation 26.26:

$$U_{\text{C}} = \frac{1}{2}C(\Delta V_C)^2 \Rightarrow \Delta V_C = \pm\sqrt{\frac{2U_C}{C}} = \pm\sqrt{\frac{2(32.5\text{ J})}{90\mu\text{F}}} = \pm0.85\text{ kV}$$

The two signs indicate that we do not know the polarity of the capacitor. However, we can say that the capacitor is charged to 0.85 kV.

Section 26.7 Dielectrics

26.35. Model: Assume the capacitor is a parallel-plate capacitor.
Solve: (a) The capacitance is increased by a factor of the dielectric constant κ over the equivalent vacuum-filled capacitor.

$$C = \kappa C_0 = \kappa\frac{\varepsilon_0 A}{d} = (2.1)\frac{(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)(0.040\text{ m})^2}{0.20\times10^{-3}\text{ m}} = 1.5\times10^{-10}\text{ F} = 0.15\text{ nF}$$

where we used $\kappa = 2.1$ for Teflon from Table 26.1.

(b) The dielectric strength is the maximum possible electric field in the capacitor before breakdown. In this capacitor,

$$E = \frac{\Delta V}{d} \Rightarrow 60\times10^6\text{ V/m} = \frac{\Delta V}{(0.20\times10^{-3}\text{ m})} \Rightarrow \Delta V = 12\text{ kV}$$

26.36. Model: The electrodes form a parallel-plate capacitor.
Solve: (a) With the battery connected, $\Delta V_{\text{C}} = 9.0\text{ V}$. The capacitor is vacuum-insulated.

$$E = E_0 = \frac{\Delta V_{\text{C}}}{d} = \frac{9.0\text{ V}}{0.10\times10^{-3}\text{ m}} = 90\text{ kV/m}$$

$$Q = C_0\Delta V_{\text{C}} = (2.21\times10^{-12}\text{ F})(9.0\text{ V}) = 20\text{ pC}$$

(b) The capacitance of the vacuum-filled capacitor is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)(5.0\times10^{-3}\text{ m})^2}{(0.10\times10^{-3}\text{ m})} = 2.21\times10^{-12}\text{ F}$$

From the table in the text, the dielectric constant of Mylar is $\kappa = 3.1$. With the battery attached, the potential difference across the plates with the Mylar inserted is still $\Delta V_{\text{C}} = \Delta V_{\text{batt}} = 9.0\text{ V}$. The charge on the plates is

$$Q = C\Delta V = \kappa C_0\Delta V = (3.1)(2.21\times10^{-12}\text{ F})(9.0\text{ V}) = 6.17\times10^{-11}\text{ C} = 62\text{ pC}$$

The electric field inside the capacitor is

$$E = \frac{E_0}{\kappa} = \frac{1}{\kappa}\frac{\Delta V_{\text{C}}}{d} = \left(\frac{1}{3.1}\right)\left(\frac{9.0\text{ V}}{0.10\times10^{-3}\text{ m}}\right) = 29\text{ kV/m}$$

Assess: Since the battery remains connected as the Mylar is inserted, the potential difference across the plates does not change.

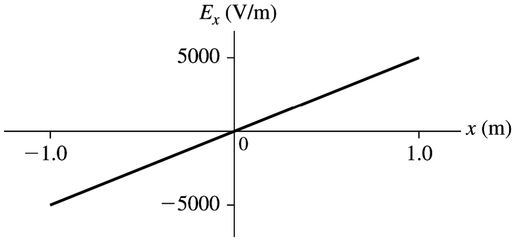
26.37. Model: Model the cell membrane as a parallel-plate capacitor and the cell as a spheroid.
Solve: For a parallel-plate capacitor, Equation 26.18 tells us that $C_0 = \varepsilon_0 A/d$. The area A is the surface area of a 50-μm-diameter sphere; $A = 4\pi r^2$. With a dielectric between the capacitor plates, the capacitance becomes $C = \kappa C_0$ (see Equation 26.32). Combining these gives

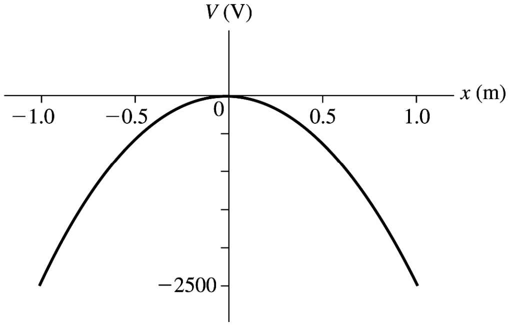
$$C = \kappa\frac{\varepsilon_0 A}{d} = \kappa\frac{4\pi r^2 \varepsilon_0}{d} = (9.0)\frac{4\pi(25\times10^{-6}\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)}{7.0\times10^{-9}\text{ m}} = 89\text{ pF}$$

Assess: This capacitance is comparable to that of typical laboratory capacitors.

Problems

26.38. Solve: (a) See plots below.





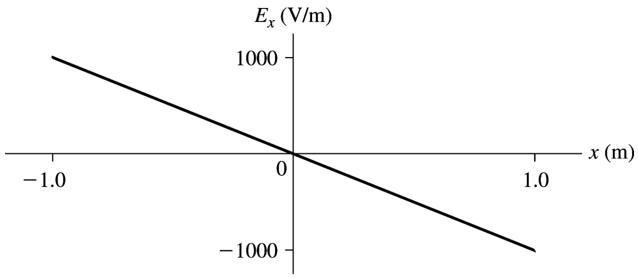
(b) Equation 26.3 gives the potential difference between two points in space:

$$\Delta V = V(x_\text{f}) - V(x_\text{i}) = -\int_{x_\text{i}}^{x_\text{f}} E_x dx = -\int_{x_\text{i}}^{x_\text{f}} (5000x\text{ V/m})dx = -5000\left[\frac{x^2}{2}\right]_{x_\text{i}}^{x_\text{f}}\text{ V} = -2500(x_\text{f}^2 - x_\text{i}^2)\text{ V}$$

Taking $V(x_\text{i}) = 0$ V at $x_\text{i} = 0$ m, and replacing x_f with simply x gives $V(x) = -(2500\,x^2)$ V.
(c) A graph of V versus x over the region $-1\text{ m} \le x \le 1\text{ m}$ is shown in part (a).
Assess: As it must be, we have

$$-\frac{dV}{dx} = -\frac{d}{dx}(-2500x^2)\text{ V} = 5000x = E_x$$

26.39. Solve: (a) See plot of $E(x) = -1000x$ V/m below.



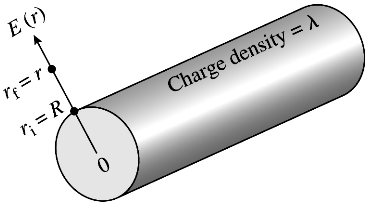
(b) Equation 26.3 gives the potential difference between two points in space:

$$\Delta V = -\int_{x_i}^{x_f} E_x dx = -\int_{-20\text{ cm}}^{30\text{ cm}} (-1000x\text{ V/m})dx$$

$$= +\left[1000\frac{x^2}{2}\right]_{-0.20\text{ m}}^{0.30\text{ m}}\text{ V} = +\frac{1000}{2}[(0.30\text{ m})^2 - (-0.20\text{ m})^2]\text{ V} = +25\text{ V}$$

Assess: E is positive for negative x but negative for positive x, so the potential difference depends on the square of the positions.

26.40. Visualize:



Solve: Equation 26.3 gives the potential difference between two points in space. With $V(R) = V_0$, this gives

$$\Delta V = V(r_f) - V(r_i) = -\int_{r_i}^{r_f} E_r dr$$

$$V(r) - V(R) = -\int_R^r \frac{\lambda}{2\pi\varepsilon_0 r}dr = -\frac{\lambda}{2\pi\varepsilon_0}[\ln r]_R^r = -\frac{\lambda}{2\pi\varepsilon_0}[\ln r - \ln R] = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{R}$$

$$V(r) = V_0 - \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{R}$$

Assess: At $r = R$, $\ln(R/R) = 0$ and $V(R) = V_0$, as it must be due to the assumed constant of integration.

26.41. Model: Assume the electrodes form parallel-plate capacitors with a uniform electric field between the plates.
Visualize: Please refer to Figure P26.41. The three metal electrodes serve as plates for two capacitors. On the middle electrode, half the charge is located on the left face and half on the right face, thus forming two capacitors. Each plate of the two capacitors thus carries a charge of ±50 nC.
Solve: **(a)** In the space $0\text{ cm} < x < 1\text{ cm}$, the magnitude of the electric field is

$$E = \frac{\eta}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{50\times10^{-9}\text{ C}}{(0.020\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{N m}^2)} = 1.41\times10^7\text{ V/m}$$

Thus, at $x = 0.5$ cm, $E = 1.4\times10^7$ V/m. Because the electric field points from positive to negative charge, the direction of the electric field is to the left, or $\vec{E} = -(1.4\times10^7\hat{i})$ V/m. The potential difference between two points in space with a uniform electric field is

$$\Delta V = V_f - V_i = E(x_f - x_i)$$

With the negative plate at $x = 0$ m defined as zero potential ($V_i = 0$ V at $x_i = 0$ cm), $V_f = x_f E$, or simply $V = xE$. At $x = 0.5$ cm, the potential is

$$V = xE = (0.5\times10^{-2}\text{ m})(1.41\times10^7\text{ V/m}) = 7\times10^4\text{ V}$$

(b) In the region $1\text{ cm} \le x \le 2\text{ cm}$, $\vec{E} = 0$ because in electrostatics there is no electric field inside a conductor. The electric potential is the same as at the surface of the capacitor plate, which, from part (a), is

$$V = xE = (1.0\times10^{-2}\text{ m})(1.41\times10^{7}\text{ V/m}) = 1.4\times10^{5}\text{ V}$$

(c) From the symmetry of the setup, the electric field at $x = 2.5\text{ cm}$ has the same magnitude as that at 0.5 cm, but points in the opposite direction, so $\vec{E} = (1.4\times10^{7}\hat{i})\text{ V/m}$. Similarly, the potential must be the same, so $V = 7\times10^{4}\text{ V}$.

26.42. Model: The electric field is the negative of the slope of the potential function.
Solve: The on-axis potential of a charged disk obtained in Chapter 28 was

$$V_{\text{disk}} = \frac{Q}{2\pi\varepsilon_0 R^2}\left[\sqrt{R^2+z^2} - z\right]$$

where the charge on the disk of radius R is Q and the point is a distance z away from the center of the disk. Because the magnitude E of the electric field is $E = -dV/dz$, we have

$$E_{\text{disk}} = -\frac{dV_{\text{disk}}}{dz} = -\frac{Q}{2\pi\varepsilon_0 R^2}\left[\frac{\frac{1}{2}(2z)}{\sqrt{R^2+z^2}} - 1\right] = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]$$

If $Q > 0$, the electric field lines will point away from the disk, so the electric field is

$$\vec{E}_{\text{disk}}(z) = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]\hat{k}$$

Assess: Using binomial expansion with $z >> R$ we have

$$E_{\text{disk}} = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{z(1+R^2/z^2)^{1/2}}\right] = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1-(1+R^2/z^2)^{-1/2}\right] = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1-1+\tfrac{1}{2}R^2/z^2+\cdots\right] = \frac{Q}{4\pi\varepsilon_0 z^2}$$

That is, the disk behaves like a point charge. This is an expected result.

26.43. Model: Model the charged rod as a line of charge of length L.
Visualize: Please refer to Figure P26.43.
Solve: (a) Divide the charged rod into N small segments, each of length Δx and with charge Δq. The segment i located at position x_{i} contributes a small amount of potential V_{i} at point P:

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 (x_0 - x_i)} = \frac{Q\Delta x/L}{4\pi\varepsilon_0 (x_0 - x_i)}$$

Point P is at a distance x_0 from the origin. This is done to avoid confusion with x_{i}. The V_{i} are now summed and the sum is converted to an integral giving

$$V = \frac{Q}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2}\frac{dx}{x_0 - x} = \frac{Q}{4\pi\varepsilon_0 L}\left[-\ln(x_0 - x)\right]_{-L/2}^{L/2} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x_0 + L/2}{x_0 - L/2}\right)$$

Replacing x_0 with x, the potential due to a line charge of length L at a distance x from the center is

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x + L/2}{x - L/2}\right)$$

(b) Because $E_x = -dV/dx$,

$$E_x = -\frac{Q}{4\pi\varepsilon_0 L}\frac{d}{dx}\left[\ln(x+L/2) - \ln(x-L/2)\right] = -\frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{(x+L/2)} - \frac{1}{(x-L/2)}\right] = \frac{Q}{4\pi\varepsilon_0}\frac{1}{x^2 - L^2/4}$$

For $Q > 0$, the electric field will point away from the rod, so

$$\vec{E}_x = \frac{Q}{4\pi\varepsilon_0}\frac{\text{sgn}(x)}{x^2 - L^2/4}\hat{i}$$

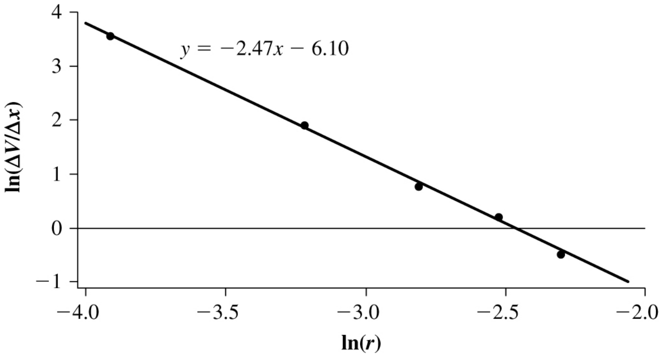
Assess: When $L = 0$ m, $E_x = Q/(4\pi\varepsilon_0 x^2)$. This is the electric field of a point charge Q a distance x away from a point charge, as expected.

26.44. Model: The electric field is the negative of the slope of the potential graph. Model the electric field of the charges as $E_r = C/r^n$.

Solve: The probe measures the potential difference ΔV over a very small distance $\Delta x = 1.00$ mm. Because the field is essentially uniform over very small distances, the magnitude of the field is well approximated by $E = \Delta V/\Delta x$. Therefore

$$E_r = \frac{\Delta V}{\Delta x} = \frac{C}{r^n} = Cr^{-n} \Rightarrow \ln(\Delta V/\Delta x) = \ln C - n \ln r$$

Thus, if we plot $\ln(\Delta V/\Delta x)$ versus ln r, the graph should be a straight line having slope equal to $-n$ and y-intercept equal to ln C.



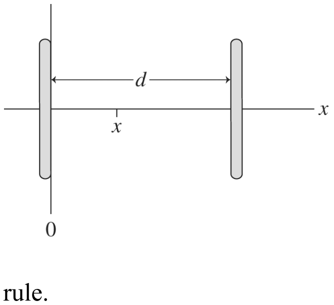
The linear fit is very good. From the experimental slope of –2.47, we see that n = 2.5 (to two significant figures). Because the y-intercept of –6.10 is equal to ln C, we have

$$\ln C = -6.10 \Rightarrow C = e^{-6.10} = 2.2\times10^{-3}\ \text{V m}^{3/2}$$

The units of C come from knowing that C divided by $\text{m}^{5/2}$ has to give electric field units of V/m.

26.45. Model: The charged electrodes form a capacitor.

Visualize: We are given $V(x) = V_0 \ln(1 + x/d)$. We seek the field strength at $x = d/2$.



Solve: Take the derivative using the chain rule.

$$E_x = -\frac{dV_x}{dx} = -\frac{d}{dx}\left(V_0 \ln\left(1 + \frac{x}{d}\right)\right) = -V_0\left(\frac{1}{1 + x/d}\right)\frac{1}{d} = -V_0\frac{1}{d + x}$$

Now evaluate at $x = d/2$ and drop the negative sign since we are looking for the field strength, not direction.

$$E = V_0\frac{1}{d + x} = V_0\frac{1}{d + d/2} = V_0\frac{1}{(3/2)d} = \frac{2V_0}{3d}$$

26.46. Model: The electric field is the negative of the slope of the graph of the potential function.
Solve: The electric potential in a region of space is $V = (150x^2 - 200y^2)$ V where x and y are in meters. The x- and y-components of the electric field are

$$E_x = -\frac{dV}{dx} = -(300\ x)\ \text{V/m} \qquad E_y = -\frac{dV}{dy} = +(400\ y)\ \text{V/m}$$

At $(x, y) = (2.0\ \text{m},\ 2.0\ \text{m}),\ E_x = -600\ \text{V/m}$ and $E_\text{y} = 800\ \text{V/m}$. The magnitude and direction of the electric field are

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-600\ \text{V/m})^2 + (800\ \text{V/m})^2} = 1000\ \text{V/m}$$

$$\tan\theta = \frac{E_y}{|E_x|} = \frac{800\ \text{V/m}}{600\ \text{V/m}} = \frac{4}{3} \Rightarrow \theta = 53°\ \text{above the } -x\text{-axis}$$

The electric field points $180° - 53° = 127°$ counterclockwise from the $+x$-axis.

26.47. Solve: The electric field is related to the electric potential by

$$\vec{E}(x, y) = -\vec{\nabla}V(x, y) = -\left(\frac{\partial V}{dx}\hat{i} + \frac{\partial V}{dy}\hat{j}\right)$$

Compute the partial derivatives:

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left(\frac{200}{\sqrt{x^2 + y^2}}\right) = 200\left(-\frac{1}{2}\right)(2x)(x^2 + y^2)^{-3/2} = -\frac{200x}{(x^2 + y^2)^{3/2}}$$

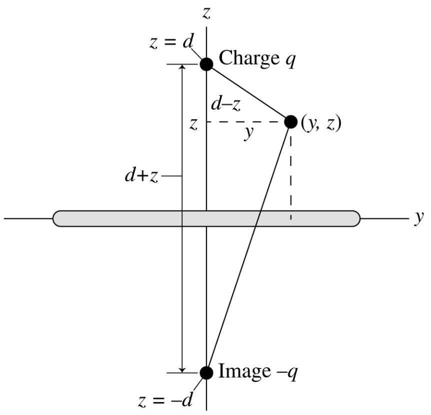
$$\frac{\partial V}{\partial y} = -\frac{200y}{(x^2 + y^2)^{3/2}}$$

Thus $\vec{E} = \frac{200}{(x^2 + y^2)^{3/2}}(x\hat{i} + y\hat{j})$. At $(x, y) = (2.0\ \text{m},\ 1.0\ \text{m})$, $\vec{E} = 17.9(2.0\hat{i} + 1.0\hat{j})$ V/m. The magnitude

$$E = 17.9\sqrt{(2.0^2 + 1.0^2)}\ \text{V/m} = 40\ \text{V/m}$$

The direction is $\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 27°$ ccw from the $+x$-axis.

26.48. Model: There is symmetry in the xy-plane.
Visualize:



Solve: (a) Add the contributions to the field from the charge q and the image charge $-q$.

$$V(y,z)=K\left[\frac{q}{\sqrt{(d-z)^2+y^2}}+\frac{-q}{\sqrt{(d+z)^2+y^2}}\right]=Kq\left[\frac{1}{\sqrt{(d-z)^2+y^2}}-\frac{1}{\sqrt{(d+z)^2+y^2}}\right]$$

(b) Use $E_z=-dV/dz$. Use the chain rule in the derivative and cancel negative signs.

$$E_z=-\frac{dV}{dz}=-Kq\left[\left(\frac{-1}{2}\right)((d-z)^2+y^2)^{-3/2}(2(d-z))(-1)-\left(\frac{-1}{2}\right)((d+z)^2+y^2)^{-3/2}(2(d+z))(1)\right]$$

$$=-Kq\left[\frac{d-x}{((d-z)^2+y^2)^{3/2}}+\frac{d+x}{((d+z)^2+y^2)^{3/2}}\right]$$

To find the field in the z-direction at the surface of the plate (there is no y-component there), set $z=0$ in the equation above. This should be a function of y only.

$$E_z\big|_{z=0}=-Kq\left[\frac{d}{(d^2+y^2)^{3/2}}+\frac{d}{(d^2+y^2)^{3/2}}\right]=-2Kq\frac{d}{(d^2+y^2)^{3/2}}$$

(c) Insert the values given into the answer above. We are given $q=10\text{ nC}$ and $d-0.020\text{ m}$.

(i)

$$E_z\big|_{z=0,y=0}=-2Kq\frac{d}{(d^2+0^2)^{3/2}}=\frac{-2Kq}{d^2}=\frac{-2(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10\text{ nC})}{(0.020\text{ m})^2}=-4.5\times10^5\ \text{V/m}$$

So the strength is 4.5×10^5 V/m.

(ii)

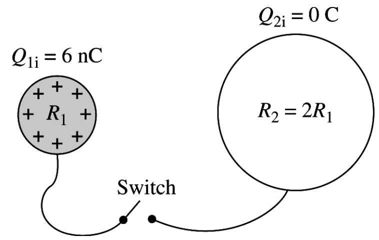
$$E_z\big|_{z=0,y=2.0\text{ cm}}=-2Kq\frac{d}{(d^2+y^2)^{3/2}}$$

$$=-2(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10\text{ nC})\frac{(0.020\text{ m})}{((0.020\text{ m})^2+(0.020\text{ m})^2)^{3/2}}=-1.6\times10^5\ \text{V/m}$$

So the strength is 1.6×10^5 V/m.

Assess: The units cancel properly. The answer 2.0 cm away from the center point has a smaller field, as we expect.

26.49. Model: Conductors connected by a conducting wire are at the same potential.
Visualize:



Charge flows from sphere 1 to sphere 2 when the wire is connected. Once connected, the entire system is one large conductor in electrostatic equilibrium. Consequently, the entire system must be an equipotential. That is, the two spheres must have equal potentials $V_1=V_2$ after being connected by the wire.

Solve: The potential of a sphere is $V=Q/(4\pi\varepsilon_0R)$, so the final charges $Q_{1\text{f}}$ and $Q_{2\text{f}}$ on the two spheres must be such that

$$\frac{Q_{1f}}{4\pi\varepsilon_0 R_1} = \frac{Q_{2f}}{4\pi\varepsilon_0 R_2} = \frac{Q_{2f}}{4\pi\varepsilon_0 (2R_1)} \Rightarrow Q_{1f} = \tfrac{1}{2}Q_{2f}$$

where we used the fact that $R_2 = 2R_1$. It is also true that charge must be conserved, so

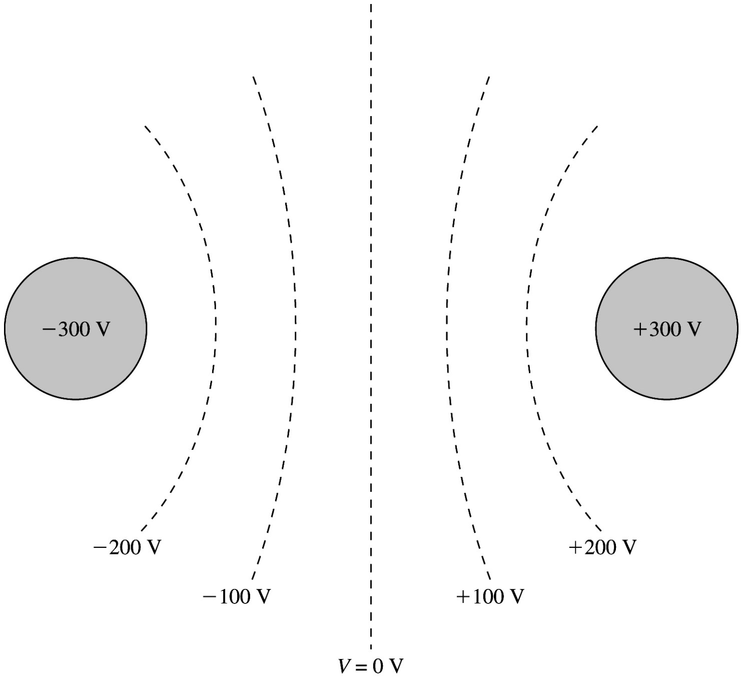
$$Q_{1f} + Q_{2f} = Q_{1i} + Q_{2i} = 6\text{ nC}$$

Using $Q_{2f} = 2Q_{1f}$, we find

$$Q_{1f} + 2Q_{1f} = 3Q_{1f} = 6\text{ nC}$$

from which we finally get $Q_{1f} = 2$ nC and $Q_{2f} = 4$ nC.

26.50. Solve:



Because the spheres have equal and opposite charges and they have the same diameter, their potential is zero on a line that bisects the two spheres and, because the spheres are of equal size and charge, the equipotential lines must be symmetric about this line. The equipotential lines at 0 V, ±100 V, and ±200 V are shown in the figure.

26.51. Model: The potential inside a conductor is the same as the potential on the surface.
Solve: The potential on the surface of the copper sphere must be 500 V. That is,

$$500\text{ V} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow Q = (4\pi\varepsilon_0)R(500\text{ V}) \Rightarrow = \frac{(0.020\text{ m})(500\text{ V})}{9.0\times10^9\text{ N m}^2/\text{C}} = 1.1\text{ nC}$$

26.52. Model: The potential due to the sphere is as if the charge were concentrated at the center.
Visualize: Let *r* be the distance from the center of the sphere to point A. $V = Kq/r$.

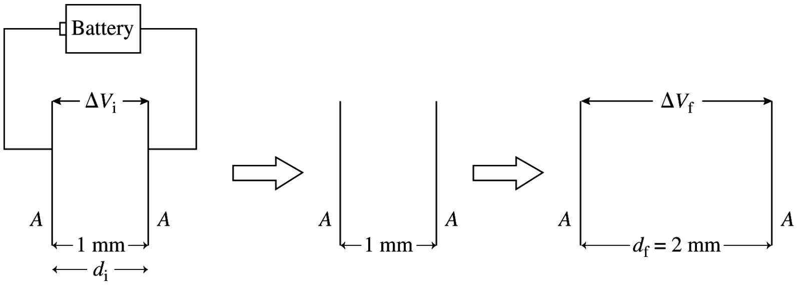
Solve: Use ratios.

$$\frac{V_A}{V_B} = \frac{Kq/r}{Kq/(r+2.0\ \mu\text{m})} = \frac{r+2.0\ \mu\text{m}}{r} = \frac{40\ \text{V}}{40\ \text{V} - 1.6\ \text{mV}} = \frac{40\ \text{V}}{39.9984\ \text{V}} \Rightarrow$$

$$r + 2.0\ \mu\text{m} = r\frac{40\ \text{V}}{39.9984\ \text{V}} \Rightarrow r = \frac{2.0\ \mu\text{m}}{\dfrac{40\ \text{V}}{39.9984\ \text{V}} - 1} = 5.0\ \text{cm}$$

Assess: This seems like a reasonable distance if the sphere is smaller than 10 cm in diameter.

26.53. Model: The battery is assumed to be ideal.
Visualize:



The pictorial representation shows the capacitor plates connected to a battery, the battery removed from the plates, and the plates moved apart with insulating handles.

Solve: (a) The battery charges the plates through the wires. Once the wires are disconnected, the charge is trapped on the plates and *will not change* as their spacing is increased. The initial capacitance of the plates is

$$C_i = \frac{\varepsilon_0 A}{d_i} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)(0.020\ \text{m}\times0.020\ \text{m})}{0.0010\ \text{m}} = 3.54\times10^{-12}\ \text{F}$$

Consequently, the initial voltage $\Delta V_i = 9.0$ V charges the plates to

$$Q = \pm C\Delta V = \pm(3.54\times10^{-12}\ \text{F})(9.0\ \text{V}) = \pm3.2\times10^{-11}\ \text{C}$$

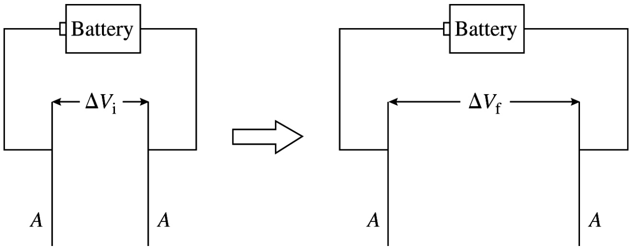
(b) The charge is still $Q = \pm3.2\times10^{-11}$ C after the spacing is increased, but the final capacitance is

$$C_f = \frac{\varepsilon_0 A}{d_f} = \frac{\varepsilon_0 A}{2d_i} = \frac{C_i}{2}$$

As a result, the final potential difference is

$$\Delta V_f = \frac{Q}{C_f} = \frac{Q}{C_i/2} = 2\frac{Q}{C_i} = 2\Delta V_i = 2\times9.0\ \text{V} = 18\ \text{V}$$

26.54. Model: Assume the battery is ideal.
Visualize:



The pictorial representation shows the capacitor plates connected to a battery, and the capacitor plates moved apart with insulating handles while the plates are connected to the battery.

Solve: **(a)** The initial capacitance of the plates is

$$C_i = \frac{\varepsilon_0 A}{d_i} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)(0.020\ \text{m})^2}{1.0\times10^{-3}\ \text{m}} = 3.54\times10^{-12}\ \text{F}$$

Consequently, an initial voltage $\Delta V_i = 9.0\ \text{V}$ charges the plates to

$$Q = \pm C_i \Delta V_i = \pm(3.54\times10^{-12}\ \text{F})(9.0\ \text{V}) = \pm31.9\times10^{-12}\ \text{C} = \pm32\ \text{pC}$$

(b) The new capacitance is $C_f = \varepsilon_0 A/2d_i = \frac{1}{2}C_i$. The potential difference across the plates is determined by the battery and is unchanged: $\Delta V_f = \Delta V_i = 9.0$. Thus, the new charge on the plates is

$$Q = \pm C_i \Delta V_f = \pm\tfrac{1}{2}(3.54\times10^{-12}\ \text{F})(9.0\ \text{V}) = \pm16.0\times10^{-12}\ \text{C} = \pm16.0\ \text{pC}$$

26.55. **Solve:** **(a)** The equivalent capacitance of *N* capacitors that are in parallel and have capacitances $C_1, C_2, C_3, \cdots, C_N$ is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

If the capacitors are identical with each having a capacitance *C*, then $C_{eq} = NC$.

(b) The equivalent capacitance of *N* capacitors that are in series and have capacitances $C_1, C_2, C_3, \cdots, C_N$ is

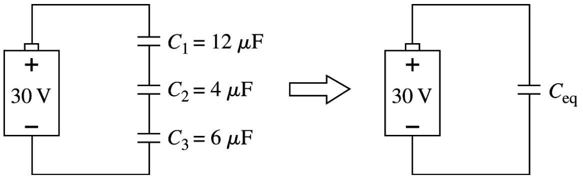
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

If the capacitors are identical with each capacitor having a capacitance *C*, then

$$\frac{1}{C_{eq}} = \frac{N}{C} \Rightarrow C_{eq} = \frac{C}{N}$$

26.56. **Model:** Assume that the battery is ideal.

Visualize:



The pictorial representation shows the equivalent capacitance of the three capacitors.

Solve: Because C_1, C_2, and C_3 are in series,

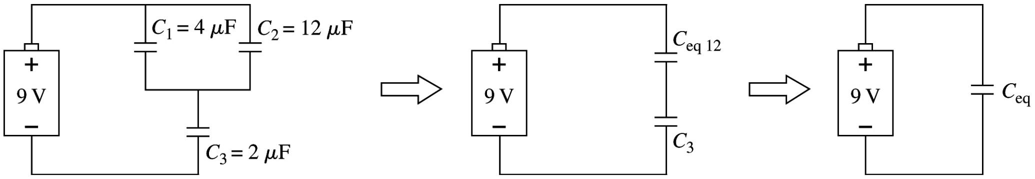
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{12\ \mu\text{F}} + \frac{1}{4\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} = \frac{1}{2}(\mu\text{F})^{-1} \Rightarrow C_{eq} = 2\ \mu\text{F}$$

A potential difference of $\Delta V_C = \varepsilon = 30\ \text{V}$ across a capacitor of equivalent capacitance 2 μF produces a charge $Q = C_{eq}\Delta V_C = (2\mu\text{F})(30\ \text{V}) = 60\mu\text{C}$. Because C_{eq} is a combination of three series capacitors, $Q_1 = Q_2 = Q_3 = 60\mu\text{C}$. We are now able to find the potential difference across each capacitor:

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{60\ \mu\text{C}}{12\ \mu\text{F}} = 5.0\ V \qquad \Delta V_2 = \frac{Q_2}{C_2} = \frac{60\ \mu\text{C}}{4\ \mu\text{F}} = 15\ \text{V} \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{60\ \mu\text{C}}{6\ \mu\text{F}} = 10\ \text{V}.$$

Assess: $\Delta V_1 + \Delta V_2 + \Delta V_3 = 30\ \text{V} = \Delta V_{bat}$, as it should.

26.57. Model: Assume that the battery is ideal.
Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.
Solve: Because C_1 and C_2 are in parallel,

$$C_{\text{eq 12}} = C_1 + C_2 = 4\mu\text{F} + 12\mu\text{F} = 16\mu\text{F}$$

$C_{\text{eq 12}}$ and C_3 are in series, so

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_{\text{eq 12}}} + \frac{1}{C_3} = \frac{1}{16\ \mu\text{F}} + \frac{1}{2\ \mu\text{F}} = \frac{18}{32}(\mu\text{F})^{-1} \Rightarrow C_{\text{eq}} = \frac{16}{9}\ \mu\text{F}$$

A potential difference of $\Delta V_{\text{C}} = 9$ V across a capacitor of equivalent capacitance $\frac{16}{9}\ \mu$F produces a charge

$$Q = C_{\text{eq}}\Delta V_{\text{C}} = \left(\tfrac{16}{9}\ \mu\text{F}\right)9\ \text{V} = 16\ \mu\text{C}$$

Because C_{eq} is a series combination of two capacitors $C_{\text{eq 12}}$ and C_3, $Q_3 = Q_{\text{eq 12}} = 16\ \mu$C. The potential difference across C_3 is

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{16\ \mu\text{C}}{2\ \mu\text{F}} = 8\ \text{V}$$

Now, $Q_{\text{eq 12}} = 16\ \mu$C is the charge on the equivalent capacitor with $C_{\text{eq 12}} = 16\ \mu$F. So, the potential difference across the equivalent capacitor $C_{\text{eq 12}}$ is

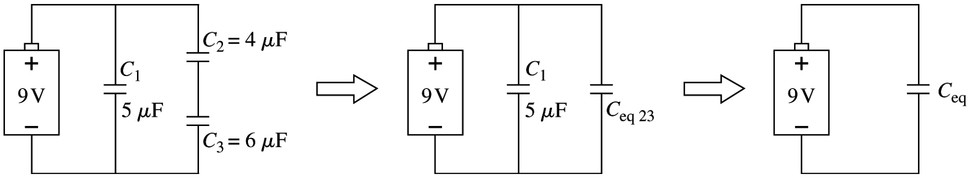
$$\Delta V_{\text{eq 12}} = \frac{Q_{\text{eq 12}}}{C_{\text{eq 12}}} = \frac{16\ \mu\text{C}}{16\ \mu\text{F}} = 1.0\ \text{V}$$

Parallel capacitors C_1 and C_2 have the same potential difference as the equivalent capacitor $C_{\text{eq 12}}$, so $\Delta V_1 = \Delta V_2 = 1.0$ V. The charge on each is given by $Q = C\Delta V$, so $Q_1 = (4\ \mu\text{F})(1.0\ \text{V}) = 4\ \mu$C and $Q_2 = (12\ \mu\text{F})(1.0\ \text{V}) = 12\ \mu$C.

In summary, $Q_1 = 4\ \mu$C, $\Delta V_1 = 1.0$ V; $Q_2 = 12\ \mu$C, $\Delta V_2 = 1.0$ V; and $Q_3 = 16\ \mu$C, $\Delta V_3 = 8$ V.

Assess: Note that $\Delta V_3 + \Delta V_{\text{eq 12}} = 9.0\ \text{V} = \Delta V_{\text{bat}}$, as it should. Also that $Q_1 + Q_2 = 16.0\ \mu\text{C} = Q_{\text{eq 12}}$, as it should.

26.58. Model: Assume the battery is an ideal battery.
Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.

Solve: Because C_2 and C_3 are in series,

$$\frac{1}{C_{\text{eq }23}}=\frac{1}{C_2}+\frac{1}{C_3}=\frac{1}{4\ \mu\text{F}}+\frac{1}{6\ \mu\text{F}}=\frac{10}{24}(\mu\text{F})^{-1}\Rightarrow C_{\text{eq }23}=\frac{12}{5}\mu\text{F}=\ 2.4\ \mu\text{F}$$

$C_{\text{eq }23}$ and C_1 are in parallel, so

$$C_{\text{eq}}=C_{\text{eq }23}+C_1=2.4\ \mu\text{F}+5\ \mu\text{F}=7.4\ \mu\text{F}$$

A potential difference of $\Delta V_\text{C}=9\text{ V}$ across a capacitor of equivalent capacitance 7.4 μF produces a charge

$$Q=C_{\text{eq}}\Delta V_\text{C}=\left(\frac{37}{5}\ \mu\text{F}\right)(9\text{ V})=\frac{333}{5}\ \mu\text{C}$$

Because C_{eq} is a parallel combination of C_1 and $C_{\text{eq }23}$, these capacitors have $\Delta V_1=\Delta V_{\text{eq }23}=\Delta V_\text{C}=9\text{ V}$. Thus the charges on these two capacitors are

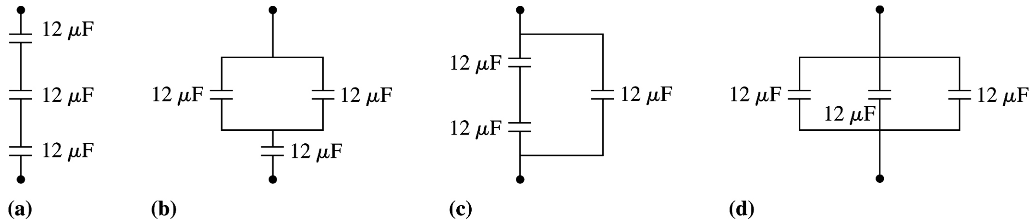
$$Q_1=(5\ \mu\text{F})(9\text{ V})=45\ \mu\text{C}\qquad Q_{\text{eq }23}=(2.4\ \mu\text{F})(9\text{ V})=21.6\ \mu\text{C}$$

Because $Q_{\text{eq }23}$ is due to a series combination of C_2 and C_3, $Q_2=Q_3=21.6\ \mu\text{C}$. This means

$$\Delta V_2=\frac{Q_2}{C_2}=\frac{21.6\ \mu\text{C}}{4\ \mu\text{F}}=\ 5.4\text{ V}\qquad \Delta V_3=\frac{Q_3}{C_3}=\frac{21.6\ \mu\text{C}}{6\ \mu\text{F}}=3.6\text{ V}$$

Thus, to two significant figures, $Q_1=45\ \mu\text{C}, V_1=9\text{ V}$; $Q_2=22\ \mu\text{C},\ V_2=5.4\text{ V}$; and $Q_3=22\ \mu\text{C}, V_3=3.6\text{ V}$.

26.59. Model: Capacitors in parallel add to a greater capacitance compared to individual capacitances. On the other hand, capacitors in series add to a smaller capacitance compared to individual capacitances.
Visualize:



Solve: **(a)** Three capacitors in series:

$$\frac{1}{C_{\text{eq}}}=\frac{1}{12\ \mu\text{F}}+\frac{1}{12\ \mu\text{F}}+\frac{1}{12\ \mu\text{F}}=\frac{3}{12}(\mu\text{F})^{-1}\Rightarrow C_{\text{eq}}=4.0\ \mu\text{F}$$

(b) Two capacitors in parallel and the third in series with this parallel combination:

$$C_{\text{eq }12}=12\ \mu\text{F}+12\ \mu\text{F}=24\ \mu\text{F}$$

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_{\text{eq }12}}+\frac{1}{12\ \mu\text{F}}=\frac{1}{24\ \mu\text{F}}+\frac{1}{12\ \mu\text{F}}=\frac{1}{8.0\ \mu\text{F}}\Rightarrow C_{\text{eq}}=8.0\ \mu\text{F}$$

(c) Two capacitors in series and the third in parallel with this series combination:

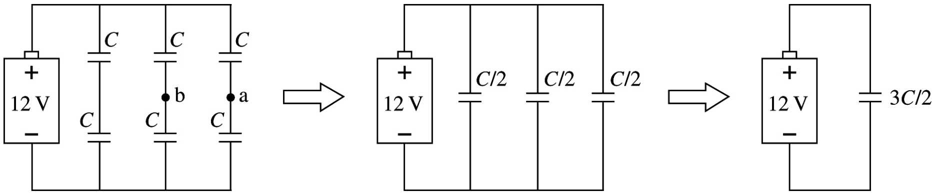
$$\frac{1}{C_{\text{eq }12}}=\frac{1}{12\ \mu\text{F}}+\frac{1}{12\ \mu\text{F}}=\frac{1}{6.0}(\mu\text{F})^{-1}\Rightarrow C_{\text{eq }12}=6.0\ \mu\text{F}$$

$$C_{\text{eq}}=C_{\text{eq }12}+12\ \mu\text{F}=6\ \mu\text{F}+12\ \mu\text{F}=18\ \mu\text{F}$$

(d) Three capacitors in parallel:

$$C_{\text{eq}}=12\ \mu\text{F}+12\ \mu\text{F}+12\ \mu\text{F}=36\ \mu\text{F}$$

26.60. Model: Assume the battery is ideal.
Visualize:



The circuit in Figure P26.60 has been redrawn to show that the six capacitors are arranged in three parallel combinations, each combination being a series combination of two capacitors.

Solve: (a) The equivalent capacitance of the two capacitors in series is $\frac{1}{2}C$. The equivalent capacitance of the six capacitors is $\frac{3}{2}C$.

(b) As points a and b are midpoints between identical capacitors, $V_\text{a} = V_\text{b} = 6.0$ V. Therefore, the potential difference between points a and b is zero.

26.61. Model: Assume the battery is ideal.
Visualize: Please refer to Figure P26.61. While the switch is in position A, the capacitors C_2 and C_3 are uncharged. When the switch is placed in position B, the charged capacitor C_1 is connected to C_2 and C_3. C_2 and C_3 are connected in series to form an equivalent capacitor $C_\text{eq 23}$.

Solve: While the switch is in position A, a potential difference of $V_1 = 100$ V across C_1 charges it to

$$Q_1 = C_1V_1 = (15\ \mu\text{F})(100\ \text{V}) = 1500\ \mu\text{C}$$

When the switch is moved to position B, this initial charge Q_1 is redistributed. The charge Q_1' goes on C_1 and the charge $Q_\text{eq 23}$ goes on $C_\text{eq 23}$. The voltage across C_1 and $C_\text{eq 23}$ is the same and $Q_1' + Q_\text{eq 23} = Q_1 = 1500\ \mu\text{C}$. Combining these two conditions, we get

$$\frac{Q_1'}{C_1} = \frac{Q_\text{eq 23}}{C_\text{eq 23}} \Rightarrow \frac{1500\ \mu\text{C} - Q_\text{eq 23}}{C_1} = \frac{Q_\text{eq 23}}{C_\text{eq 23}}$$

Since $C_\text{eq 23} = \left(\frac{1}{20\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}}\right)^{-1} = 12\ \mu\text{F}$, we can rewrite this equation as

$$\frac{1500\ \mu\text{C} - Q_\text{eq 23}}{15\ \mu\text{F}} = \frac{Q_\text{eq 23}}{12\ \mu\text{F}} \Rightarrow Q_\text{eq 23} = 0.67\ \text{mC} \Rightarrow Q_1' = Q_1 - Q_\text{eq 23} = 1.500\ \text{mC} - 0.67\ \text{mC} = 0.83\ \text{mC}$$

Having found the charge $Q_\text{eq 23,}$ it is easy to see that $Q_2 = Q_3 = 0.67$ mC because $C_\text{eq 23}$ is a series combination of C_2 and C_3. Thus,

$$\Delta V_1 = \frac{Q_1'}{C_1} = \frac{830\ \mu\text{C}}{15\ \mu\text{F}} = 55\ \text{V} \qquad \Delta V_2 = \frac{Q_2}{C_2} = \frac{670\ \mu\text{C}}{20\ \mu\text{F}} = 34\ \text{V} \qquad \Delta V_3 = \frac{Q_3}{C_3} = \frac{670\ \mu\text{C}}{30\ \mu\text{F}} = 22\ \text{V}$$

26.62. Model: Assume the battery is an ideal battery.
Visualize: Please refer to Figure P26.62. The battery is connected to two series capacitors C_1 and C_2.

Solve: The equivalent capacitance is

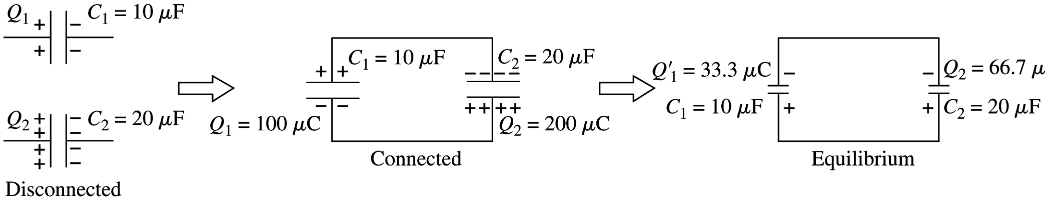
$$\frac{1}{C_\text{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_\text{eq} = \frac{C_1C_2}{C_1 + C_2}$$

Because the charge on capacitor C_2 is 450 μC, the charge on C_{eq} and C_1 is also 450 μC. This gives

$$\Delta V_{\text{eq}} = \frac{Q_{\text{eq}}}{C_{\text{eq}}} \Rightarrow 60\text{ V} = \frac{450\ \mu\text{C}}{C_1C_2/(C_1+C_2)} = \frac{(450\ \mu\text{C})(12\ \mu\text{F}+C_2)}{(12\ \mu\text{F})C_2} \Rightarrow C_2 = \frac{(450\ \mu\text{C})(12\mu\text{F})}{720\ \mu\text{C} - 450\ \mu\text{C}} = 20\ \mu\text{F}$$

Assess: Note that capacitors connected in series have the same charge.

26.63. Model: Assume the battery is ideal.
Visualize:



Solve: When the capacitors are individually charged, their charges are

$$Q_1 = C_1V = (10\ \mu\text{F})(10\text{ V}) = 100\ \mu\text{C} \qquad Q_2 = C_2V = (20\ \mu\text{F})(10\text{ V}) = 200\ \mu\text{C}$$

These two capacitors are then connected with the positive plate of C_1 connected to the negative plate of C_2, as shown in the figure above. Let the new charges on C_1 and C_2 be Q_1' and Q_2'. Then,

$$Q_1' + Q_2' = 200\ \mu\text{C} - 100\ \mu\text{C} = 100\ \mu\text{C}$$

The voltages across parallel capacitors are the same, so

$$\Delta V_1' = \Delta V_2' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \quad \Rightarrow \quad Q_2' = \frac{C_2}{C_1}Q_1'$$

Substituting this expression for Q_2' into the previous equation gives

$$Q_1' + \frac{C_2}{C_1}Q_1' = 100\ \mu\text{C} \Rightarrow Q_1'\left[1 + \frac{20\ \mu\text{F}}{10\ \mu\text{F}}\right] = 100\ \mu C$$

Solving these equations, we get $Q_1' = 33\ \mu$C and $Q_2' = 67\ \mu$C. Finally,

$$\Delta V_1' = \frac{Q_1'}{C_1} = \frac{33\ \mu\text{C}}{10\ \mu\text{F}} = 3.3\ V = \Delta V_2'$$

26.64. Solve: The magnitude of the work done by the external force is equal to the change in the electric potential energy of the capacitor. The work done is

$$W_{\text{force}} = \Delta U = U_2 - U_1 = \frac{1}{2}\frac{Q^2}{C_2} - \frac{1}{2}\frac{Q^2}{C_1}$$

Note that the charge on the plates is not changed as the distance between the electrodes changes. Thus,

$$W_{\text{force}} = \tfrac{1}{2}Q^2\left[\frac{1}{C_2} - \frac{1}{C_1}\right] = \tfrac{1}{2}(4.0\times10^{-3}\text{ C})^2\left[\frac{1}{2.0\ \mu\text{F}} - \frac{1}{5.0\ \mu\text{F}}\right] = 2.4\text{ J}$$

Assess: The work done on the capacitor is stored as electric potential energy in the capacitor.

26.65. Model: Assume each plate has a charge *Q*.

Visualize: The field due to one plate is $E = \dfrac{\eta}{2\varepsilon_0} = \dfrac{Q}{2\varepsilon_0 A}$.

Solve: (a) The force on one plate is $F = QE$ where *Q* is the charge on the one plate and *E* is the field due to the other plate. Insert *E* from above into the force equation. Then use $Q = C(\Delta V_C)$ and finally $C = \varepsilon_0 A/d$.

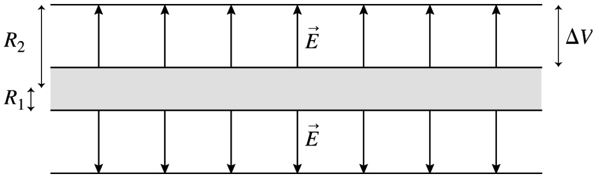
$$F = \frac{Q^2}{2\varepsilon_0 A} = \frac{C^2(\Delta V_C)^2}{2\varepsilon_0 A} = \frac{C(\Delta V_C)^2}{2d}$$

(b) Insert the values given.

$$F = \frac{C(\Delta V)^2}{2d} = \frac{(10\text{ pF})(1000\text{ V})^2}{2(1.0\text{ mm})} = 5.0\text{ mN}$$

Assess: This is a small force but is in the right range.

26.66. Visualize: For infinitely long cylinders, the electric field is radial from the inner cylinder to the outer cylinder.



Solve: (a) The electric field in the space between the cylinders is the electric field of the charged inner cylinder. (Gauss's law tells us that the outer cylinder does not contribute to the electric field between the cylinders.) From Chapter 26, we know that the field of a cylinder with linear charge density λ is

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

The potential difference between the two cylinders is

$$\Delta V = -\int_{R_1}^{R_2} E_r\,dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}[\ln(r)]_{R_1}^{R_2} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_2}{R_1}$$

The minus sign tells us that the potential decreases upon moving outward, but we only need the absolute value of ΔV. For a parallel-plate capacitor, capacitance is defined as $C = Q/\Delta V_C$. Consider a segment of the cylinders of length L. The charge on the inner cylinder is $Q = \lambda L$. Consequently, the capacitance of this segment is

$$C = \frac{Q}{\Delta V_C} = \frac{\lambda L}{(\lambda/2\pi\varepsilon_0)\ln(R_2/R_1)} = \frac{2\pi\varepsilon_0 L}{\ln(R_2/R_1)}$$

The capacitance per meter is found by dividing out the L:

$$\text{capacitance per meter} = \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(R_2/R_1)}$$

(b) Using the above expression, the capacitance per meter of cable is

$$\frac{4\pi\varepsilon_0}{2\ln(R_2/R_1)} = \frac{1}{2(9.0\times10^9\text{ N m}^2/\text{C}^2)\ln\left(\dfrac{3.0\text{ mm}}{0.50\text{ mm}}\right)} = 31\text{ pF/m}$$

26.67. Model: Use conservation of energy.

Solve: The energy stored in the capacitor is dissipated through the flash lamp. Power is the rate at which energy is dissipated or absorbed, so the energy dissipated by the flash lamp is

$$P\Delta t = (10\text{ W})(10\ \mu\text{s}) = 1.0\times10^{-4}\text{ J}$$

This is the electric potential energy in the capacitor. Using $U_C = \frac{1}{2}C\Delta V_C^2$,

$$1.0\times10^{-4}\text{ J} = \tfrac{1}{2}C(3.0\text{ V})^2 \Rightarrow C = 22\ \mu\text{F}$$

26.68. Model: Assume each plate in both capacitors has the same charge Q.
Visualize: Since the two capacitors are in series the charge on each one is the same. Use $Q = C(\Delta V)$. If the voltage across the unknown capacitor is 6.0 V, then the voltage across the 10 μF capacitor must be 3.0 V.
Solve: Apply $Q = C(\Delta V)$ to each capacitor.

$$Q = (10\ \mu F)(3.0\text{ V}) \text{ and } Q = C(6.0\text{ V}) \Rightarrow C = \frac{(10\ \mu F)(3.0\text{ V})}{(6.0\text{ V})} = 5.0\ \mu\text{F}$$

Assess: This is a reasonable value for a capacitor.

26.69. Model: Use the letter I for current.
Visualize: Remember $Q = C(\Delta V)$.
Solve: Take the derivative.

$$I = \frac{dQ}{dt} = \frac{d}{dt}C(\Delta V) = C\frac{dV}{dt} = (10\ \mu\text{F})(2.0\text{ V/s}) = 20\ \mu\text{F}\cdot\text{V/s} = 20\ \mu\text{C/S}$$

Assess: This is a typical value for a current. The unit of current is the ampere. A = FV/s.

26.70. Model: The capacitor is storing energy at a constant 200 W rate.
Visualize: After 2.0 μs the capacitor has stored $U_\text{C} = (200\text{ W})\cdot(2.0\ \mu\text{s}) = 4.0\times10^{-4}$ J.
Solve:

$$U = \tfrac{1}{2}C(\Delta V_\text{C})^2 \Rightarrow \Delta V_\text{C} = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(4.0\times10^{-4}\text{ J})}{2.0\ \mu\text{F}}} = 20\text{ V}$$

Assess: This is a typical value for the voltage across a capacitor.

26.70. Model: The capacitor is storing energy at a constant 200 W rate.

Visualize: After 2.0 μs the capacitor has stored $U_\text{C} = (200\text{ W})\cdot(2.0\ \mu\text{s}) = 4.0\times10^{-4}$ J.

Solve: Solve for q at 2.0 μs.

$$U_\text{C} = \frac{1}{2}\frac{q^2}{C} \Rightarrow q = \sqrt{2CU_\text{C}} = \sqrt{2(2.0\ \mu\text{F})(4.0\times10^{-4}\text{ J})} = 4.0\times10^{-5}\text{ C}$$

Now take the derivative of the above equation with respect to t and note $dq/dt = I$.

$$\frac{dU_\text{C}}{dt} = \frac{q}{C}\frac{dq}{dt} = \frac{q}{C}I \Rightarrow I = \frac{C(dU_\text{C}/dt)}{q} = \frac{(2.0\ \mu\text{F})(200\text{ W})}{4.0\times10^{-5}\text{ C}} = 10\text{ A}$$

Assess: This is a large current but it decreases quickly.

26.71. Model: Model the cell membrane as a parallel-plate capacitor and the cell as a sphere.
Solve: The energy stored in the electric field in the cell membrane is $U_\text{c} = C(\Delta V_c)^2/2$. The electric field is $E = \Delta V/d$, where $\Delta V = -70\text{ mV}$ is the potential difference across the membrane, d = 7.0 nm is the membrane thickness, and the dielectric constant of the membrane is $\kappa = 9.0$. Combining these equations and treating the cell membrane as a parallel place capacitor gives

$$C = \kappa\frac{\varepsilon_0 A}{d} = \kappa\varepsilon_0 4\pi r^2/d$$

The total energy stored in the cell membrane is

$$U_\text{C} = \frac{1}{2}C(\Delta V_\text{C})^2 = \frac{1}{2}\kappa\varepsilon_0 4\pi r^2(\Delta V_\text{C})^2/d = \kappa\varepsilon_0 2\pi r^2(\Delta V_\text{C})^2/d$$

$$U_\text{C} = (9.0)(8.85\times10^{-12}\text{ C}^2/\text{Nm}^2)(2\pi)(25\times10^{-6}\text{ m})^2(70\times10^{-3}\text{ V})^2/(7.0\times10^{-9}\text{ m}) = 2.2\times10^{-13}\text{ J}$$

Assess: This is a very small amount of energy, but remember that an individual cell is extremely small.

26.72. Model: Model the cell membrane as a parallel-plate capacitor and the cell as a spheroid.
Solve: In Problem 26.37, we found that the cell-membrane capacitance is 89 pF. To change the potential across such a capacitor from $V_\text{i} = -70$ mV to $V_\text{f} = +40$ mV requires a charge

$$Q = C(V_\text{f} - V_\text{i})$$

Each sodium ion contributes one elementary charge, so the total number *N* of sodium ions required is

$$N = \frac{Q}{e} = \frac{C(V_\text{f} - V_\text{i})}{e} = \frac{(89\text{ pF})[40\text{ mV} - (-70\text{ mV})]}{1.602\times10^{-19}\text{ C}} = 6.1\times10^7$$

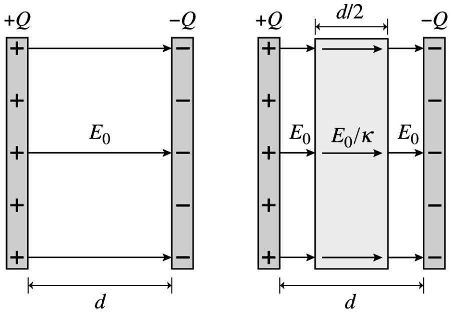
26.73. Model: The induced charge on the polarized dielectric material forms a capacitor.
Visualize: Please refer to Figure 26.31.
Solve: From Equation 26.29,

$$E_\text{induced} = E_0 - E.$$

The direction of the fields is taken care of by the signs in the equation and are as shown in Figure 26.31. Multiplying through by ε_0 and using $\kappa = E_0/E$,

$$\varepsilon_0 E_\text{induced} = \varepsilon_0 E_0 - \varepsilon_0 E = \varepsilon_0 E_0\left(1 - \frac{1}{\kappa}\right) \Rightarrow \eta_\text{induced} = \eta_0\left(1 - \frac{1}{\kappa}\right)$$

26.74. Model: The dielectric is slipped in parallel to the parallel-plate capacitor.
Visualize:



Solve: The vacuum-insulated parallel-plate capacitor has a potential difference $\Delta V_\text{C} = E_0 d$ and a capacitance $C_0 = Q/\Delta V_\text{C} = Q/(E_0 d)$. When the dielectric is placed between the plates, the electric field strength in the dielectric is reduced by a factor of κ. Thus

$$\Delta V_C' = -\int_{-\text{plate}}^{+\text{plate}} \vec{E}_0 \cdot d\vec{s} = E_0\frac{d}{4} + \frac{E_0}{\kappa}\frac{d}{2} + E_0\frac{d}{4} = E_0\frac{d}{2} + \frac{E_0}{\kappa}\frac{d}{2} = \frac{E_0 d}{2}\left(1 + \frac{1}{\kappa}\right).$$

The capacitance is now

$$C = \frac{Q}{\Delta V_\text{C}'} = \frac{Q}{E_0 d}\left(\frac{2}{1 + \frac{1}{\kappa}}\right) = C_0\frac{2\kappa}{1+\kappa}$$

Assess: For materials other than vacuum, $\kappa > 1$, and $C > C_0$ as expected. In the case of vacuum, $\kappa = 1$ and we get $C = C_0$, as expected. Note that the integral to find the potential difference is taken from the negative plate to the positive plate. While the dielectric placement was treated as though it were dead center between the plates, this is not necessary. Because half the horizontal distance between the plates is taken up by dielectric and the other half by vacuum, the integral has the same result regardless of the placement of the dielectric.

26.75. Solve: (a) Find an expression for the electric potential in a region where $E_z = 2az$ V/m where a is a constant with units of V/m^2. The potential at $z = 0$ m is 10 V.

(b) Integrating both sides with respect to z gives

$$V(z) = -a\int 2z\;dz + \text{constant} = -az^2 + \text{constant}$$

At $z = 0$ m, $V_z = 10$ V. This means the constant =10 V. Thus $V(z) = (10 - az^2)$ V.

26.76. Solve: (a) A capacitor is constructed with two 10 cm × 10 cm plates. When the capacitor is connected to a 100 V source, ±400 nC is observed on the plates. What is the separation of the plates?

(b) From the equation $Q = CV$,

$$C = \frac{400\times10^{-9}\text{ C}}{100\text{ V}} = 4.0\times10^{-9}\text{ F}$$

Thus, the second equation becomes

$$4.0\times10^{-9}\text{ F} = \frac{(8.85\times10^{-12}\text{ C}^2\text{/N m}^2)(0.10\text{ m})^2}{d} \Rightarrow d = 2.2\times10^{-5}\text{ m} = 22\ \mu\text{m}$$

26.77. Solve: (a) What capacitor should be placed parallel to 3 μF and 6 μF capacitors that are in series so that the equivalent capacitance is 4 μF?

(b) We have

$$\frac{18}{9}\mu\text{F} + C = 4\ \mu\text{F} \Rightarrow C = 2\ \mu\text{F}$$

Challenge Problems

26.78. Model: The disks form a parallel-plate capacitor.

Solve: (a) The charge on each of the metal disks is

$$Q = C\Delta V_\text{C} = \kappa_\text{Pyrex}C_0\Delta V_\text{C} = \kappa_\text{Pyrex}\frac{\varepsilon_0 A}{d}\Delta V_\text{C}$$

The charge density on a disk is

$$\eta = \frac{Q}{A} = \kappa_\text{Pyrex}\frac{\varepsilon_0\Delta V_C}{d} = (4.7)\frac{(8.85\times10^{-12}\text{ C}^2\text{/Nm}^2)(1000\text{ V})}{(0.50\times10^{-3}\text{ m})}$$

$$= 8.3\times10^{-5}\text{ C/m}^2 = 83\ \mu\text{C/m}^2$$

(b) In the absence of the dielectric, the surface charge density on the disks is

$$\eta_0 = \frac{\eta}{\kappa_\text{Pyrex}} = 1.77\times10^{-5}\text{ C/m}^2$$

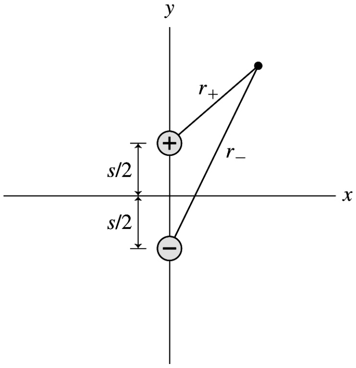
The surface charge density on the Pyrex is

$$\eta_\text{induced} = \eta_0\left(1 - \frac{1}{\kappa_\text{Pyrex}}\right) = (1.77\times10^{-5}\text{ C/m}^2)\left(1 - \frac{1}{4.7}\right)$$

$$= 1.39\times10^{-5}\text{ C/m}^2 = 14\ \mu\text{C/m}^2$$

Assess: The induced charge density on the dielectric is less than the charge density on the plates.

26.79. Visualize: The distances from the point to the two charges are r_+ and r_-. These can be expressed in terms of the coordinates x and y by using the Pythagorean theorem.



Solve: **(a)** Potential is a scalar, so the potential at (x, y) is the sum of the potentials due to each charge. This is

$$V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0 r_+} + \frac{-q}{4\pi\varepsilon_0 r_-} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2+(y-s/2)^2}} - \frac{1}{\sqrt{x^2+(y+s/2)^2}}\right]$$

(b) The binomial approximation is $(1+u)^n \approx 1 + nu$ if $u \ll 1$. To use this, we'll need to rearrange the square roots of V to be in this form. The first term is

$$\frac{1}{\sqrt{x^2+(y-s/2)^2}} = \frac{1}{\sqrt{x^2+y^2-sy+s^2/4}} = \frac{1}{\sqrt{x^2+y^2}}\frac{1}{\sqrt{1+\dfrac{-sy+s^2/4}{x^2+y^2}}}$$

$$= \frac{1}{\sqrt{x^2+y^2}}\left[1+\frac{-sy+s^2/4}{x^2+y^2}\right]^{-1/2} \approx \frac{1}{\sqrt{x^2+y^2}}\left[1-\frac{1}{2}\frac{-sy+s^2/4}{x^2+y^2}\right]$$

$$\approx \frac{1}{\sqrt{x^2+y^2}}\left[1+\frac{1}{2}\frac{sy}{x^2+y^2}-\frac{1}{8}\frac{s^2}{x^2+y^2}\right]$$

The approximation for the second term yields the same expression, except the term that is linear in y is negative. Substituting into the expression for the potential in part (a) gives

$$V = \frac{qsy}{4\pi\varepsilon_0(x^2+y^2)^{3/2}}$$

This was a lot of algebra, but the resulting expression for V is much simpler than the exact expression of part (a).

(c) The electric field components are

$$E_x = -\frac{dV}{dx} = -\frac{qsy}{4\pi\varepsilon_0}\left[-\frac{3}{2}\frac{2x}{(x^2+y^2)^{5/2}}\right] = \frac{qs(3xy)}{4\pi\varepsilon_0(x^2+y^2)^{5/2}}$$

$$E_y = -\frac{dV}{dy} = -\frac{qs}{4\pi\varepsilon_0(x^2+y^2)^{3/2}} - \frac{qsy}{4\pi\varepsilon_0}\left[-\frac{3}{2}\frac{2y}{(x^2+y^2)^{5/2}}\right] = \frac{qs(2y^2-x^2)}{4\pi\varepsilon_0(x^2+y^2)^{5/2}}$$

(d) On-axis means on the axis of the dipole, or for $x = 0$ m. In this case, $E_x = 0$ V/m and

$$E_y = \frac{qs}{4\pi\varepsilon_0}\frac{2}{y^3} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{y^3}$$

If we let $y \to r$ and note that the electric field will point away from the positive-charge dipole, we recover Equation 26.11:

$$\vec{E}_{\text{on-axis}} = E_x\hat{i} + E_y\hat{j} = \frac{2p}{4\pi\varepsilon_0 r^3}\hat{j} = \vec{E}_{\text{dipole}}$$

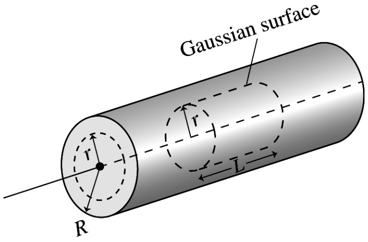
(e) The bisecting axis is the x-axis, so $y = 0$ m. In this case, $E_x = 0$ V/m and

$$E_y = -\frac{qs}{4\pi\varepsilon_0 x^3} = -\frac{p}{4\pi\varepsilon_0 x^3}$$

If we let $x \to r$ we recover Equation 26.12:

$$E_{\text{bisecting axis}} = E_x + E_y = -\frac{p}{4\pi\varepsilon_0 r^3}\hat{j} = \vec{E}_{\text{dipole}}$$

26.80. Visualize:



Solve: The charge density inside the cylinder is ρ. Consider a Gaussian surface with a radius r inside the cylinder. The charge enclosed by this closed surface is $Q_{\text{enclosed}} = (\pi r^2 L)\rho$. Using Gauss's law,

$$\oint \vec{E}\cdot d\vec{A} = E(2\pi rL) = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{(\pi r^2 L)\rho}{\varepsilon_0} \Rightarrow E = \frac{\pi r^2 L\rho}{\varepsilon_0 2\pi rL} = \left(\frac{\rho}{2\varepsilon_0}\right)r \Rightarrow \vec{E} = \left(\frac{\rho}{2\varepsilon_0}\right)r\hat{r}.$$

In the last step, we have taken into account the radial direction of the electric field pointing outward. The potential difference between the surface and the axis is

$$\Delta V = -\int \vec{E}\cdot d\vec{r} = -\int_0^R \frac{\rho}{2\varepsilon_0} r\,\hat{r}\cdot d\vec{r} = -\int_0^R \frac{\rho}{2\varepsilon_0} r\,dr = \frac{-\rho}{2\varepsilon_0}\frac{r^2}{2}\bigg|_0^R = -\frac{\rho R^2}{4\varepsilon_0}$$

26.81. Solve: (a) From Equation 26.3, the potential difference between the center (V_0) and a point at radius r is

$$\Delta V = V_r - V_0 = -\int E_r dr = -\int_0^r \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}dr = -\frac{Q}{4\pi\varepsilon_0 R^3}\frac{r^2}{2}$$

We can find V_0 from the reference point for the potential. At $r = R$ the potential matches the known outside potential, so

$$V_R = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Using the expression in the ΔV equation,

$$V_{\text{R}} - V_0 = -\frac{Q}{4\pi\varepsilon_0 R^3}\frac{R^2}{2} \Rightarrow V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2R}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{2R}\right)$$

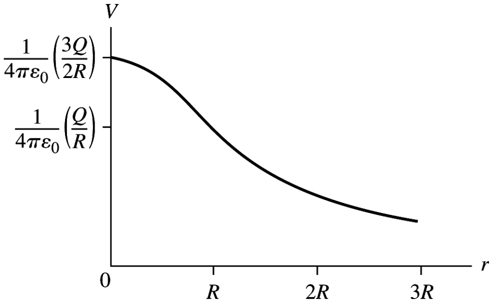
Using this result for V_0, the potential at radius r inside the sphere is

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{2R}\right) - \frac{Q}{4\pi\varepsilon_0}\left(\frac{r^2}{2R^3}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$

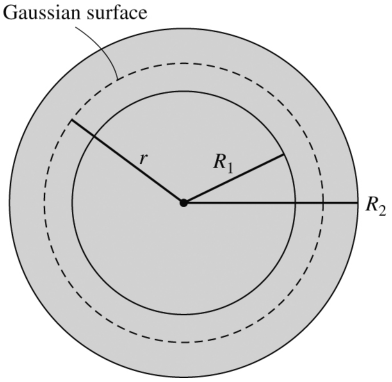
(b) The ratio is

$$\frac{V_{\text{center}}}{V_{\text{surface}}}=\frac{V_0}{V_{\text{R}}}=\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{2R}\right)\right]\Bigg/\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right)\right]=\frac{3}{2}$$

(c) A V-versus-r graph is shown in the figure.



26.82. Model: Assume $R_1 >> R_2 - R_1$ so that the capacitor may be treated as a parallel-plate capacitor.
Visualize:



Solve: **(a)** Apply Gauss's Law to the Gaussian surface shown in the figure above;

$$\oint \vec{E}\cdot d\vec{A}=\frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

The spherical symmetry of the problem means that the electric field is everywhere perpendicular to the surface element dA, so it is parallel to the surface normal $d\vec{A}$. In addition, the electric field is constant over the Gaussian surface. Combining these observations allows us to remove the electric field from the integral, which leads to

$$E\oint dA=\frac{Q_{\text{enclosed}}}{\varepsilon_0}\Rightarrow E(4\pi r^2)=\frac{Q_{\text{enclosed}}}{\varepsilon_0}\Rightarrow E=\frac{Q}{4\pi\varepsilon_0 r^2}$$

where we have used the fact that the enclosed charge is the charge on the inner capacitor surface, so $Q_{\text{enclosed}}=Q$. From Equation 26.9, we know that the electric field is the negative of the spatial derivative of the electric potential, or $E=-dV/dr$. If we integrate this in the radial direction from R_1 to R_2 we find

$$\int_{V(R_1)}^{V(R_2)} dV=-\int_{R_1}^{R_2}\frac{Q}{4\pi\varepsilon_0 r^2}dr\Rightarrow \Delta V_{\text{C}}=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where $\Delta V_{\text{C}}=V(R_2)-V(R_1)$ is the change in potential across the capacitor. Using the definition of capacitance for a parallel-plate capacitor Equation 26.14 gives the capacitance of the concentric-sphere capacitor:

$$C = \frac{Q}{\Delta V_C} = 4\pi\varepsilon_0\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1}$$

(b) Let $d = 1.0$ mm be the gap. The radii of the concentric spheres are related by $d = R_2 - R_1$. Using this and the given capacitance $C = 100$ pF gives

$$C = 4\pi\varepsilon_0\left(\frac{1}{R_1} - \frac{1}{d + R_1}\right)^{-1} \Rightarrow R_1 = \frac{1}{2}\left(-d \pm \sqrt{d^2 + \frac{dC}{\pi\varepsilon_0}}\right)$$

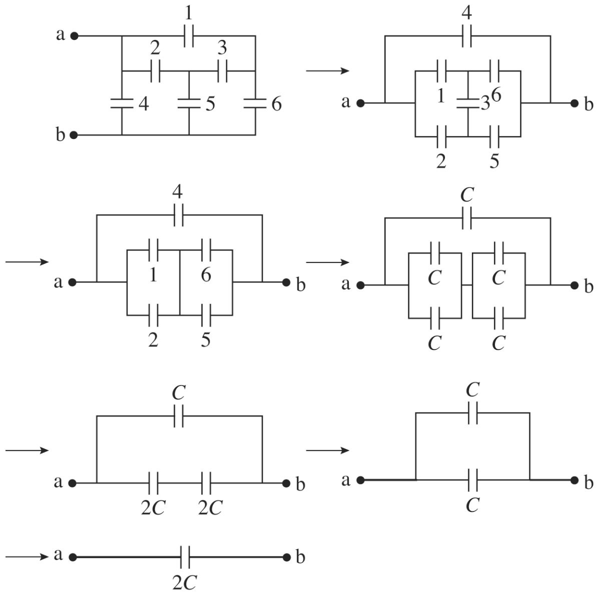
Because $R_1 > 0$, we use the positive sign of the radical, which gives

$$R_1 = \frac{1}{2}\left[-(1.0\times10^3\ \text{m}) \pm \sqrt{(1.0\times10^3\ \text{m})^2 + \frac{(1.0\times10^3\ \text{m})(1.00\times10^{-10}\ \text{F})}{\pi(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)}}\right] = 2.95\ \text{cm}$$

The radius of the outer shell is $R_2 = R_1 + d = 2.95\ \text{cm} + 1.0\ \text{mm} = 3.05\ \text{cm}$. The diameters are therefore $D_1 = 5.9$ cm, $D_2 = 6.1$ cm.

Assess: The condition $R_1 >> R_2 - R_1$ is satisfied, so we are justified in modeling this capacitor as a parallel-plate capacitor. Also, note that the expression for capacitance depends only on geometrical factors, as expected.

26.83 Visualize:



Solve: The circuit is redrawn to better visualize the series and parallel connections. The numbers help follow the rearrangement. The second part of the figure cannot be reduced by simple series and parallel reductions, but because of symmetry there can't be any potential difference across capacitor 3; hence it can be replaced with a wire and then the regular rules apply from there. The reduction to the equivalent capacitance of $2C$ is shown.